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SLOWING DOWN OF A SUPERSONIC FLOW IN A RECTANGULAR CHANNEL OF CONSTANT CROSS SECTION

M. G. Ktalkherman, V. M. Mal'kov, and N. A. Ruban

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A number of papers [2 and references in it] have been devoted to the study of the transition region from supersonic to subsonic flow in a channel [1]. Flow in this region, called the pseudoshock, is a complex process involving the interaction of shock waves with the dissipative boundary region, the details of which are not completely clear. In view of this, the main efforts have been concentrated on obtaining the "integral" characteristics of the pseudoshock, since they are of the greatest interest for practical applications.

Most papers have treated flow in circular and square pipes. Information on the efficiency of supersonic diffusers of other geometric shapes is meager and frequently contradicts the concepts which have accumulated on the pseudoshock. Relatively recent experiments showed that rectangular channels have a low efficiency of pressure recovery, particularly for large ratios of the sides. Thus, in [3-7] the increase of pressure during the slowing down of a supersonic flow was 1.3-2 times smaller than in [1]. The reasons for such a large decrease in efficiency are not clear. For rectangular channels there are no such rather general relations between the length of the pseudoshock and the ratio of the sides.

The present work was undertaken to supplement the information on the characteristics of a supersonic diffuser of constant area with a rectangular cross section, and to analyze the available experimental results.

1. The experiments were performed in a wind tunnel. Cold dry air from tanks was admitted into a forechamber with a flattening screen and through a supersonic nozzle into a rectangular channel connected to an ejector. The channel was made up of four sections with an overall length of ~1400 mm. It has a height $h = 21$ mm and a width $b = 80$ mm; $B = b/h = 3.8$. To study the effect of the ratio of the sides on the characteristics of the pseudoshock, diaphragms were used to partition the working part into three channels 21 × 25 mm in cross section ($B = 1.2$) or two channels 9 × 80 mm in cross section ($B = 8.9$). The diaphragms were located 25 mm from the exit cross section of the nozzle. Their leading and trailing edges were wedge-shaped with a half-angle of 10°.

The upper (wide) wall, which was an extension of the profiled wall of the nozzle, had vent holes along the center line. The pressure on the wall was measured, and Stanton tubes were used to determine the profile of the total head and the static pressure at several cross sections of the channel.

Three supersonic nozzles of rectangular cross section and two sets of axisymmetric nozzles (honeycomb nozzles) were used in the experiments. The basic parameters of the nozzles are listed in Table 1, where h^* is the height of the critical cross section, l_s is the length of the supersonic part of the nozzle, and A/A^* is the ratio of the exit and critical cross sectional areas. Nozzles I and II were of minimum length, and the sharp edge in the region of the critical cross section was rounded off to a radius of ~0.5 mm. The supersonic contour of nozzle III was completed along the arc of a circle, and at the exit the direction of the

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TABLE 1

Nozzle No.	h^*, mm	h, mm	l_s, mm	A/A^*
I	0,7	21	56	30
II	0,4	21	56	52,5
III	0,5	21	45	42,5

walls coincided with the direction of flow. In the transonic region in all the versions there was a rectilinear portion ~0.2 mm long. The subsonic part of nozzles I and III were tapered with a half-angle of 60°, and nozzle II has a contour profiled according to the Vitoshinskii formula.

The honeycomb nozzles consisted of a system of small openings closely spaced in a plate. In the first version conical nozzles with an apex half-angle of 10°, a critical diameter d^* = 1.1 mm, and an exit diameter d = 5.7 mm were arranged in four rows in a staggered order. The area at the edge was 26.8 times larger than the critical area, and the ratio of the area of the working channel (21 × 80 mm) to the total area of the critical cross sections was 35.3. In the second version the number of nozzles and their arrangement was the same, but their supersonic part had a profile of minimum length, and the angle between the contour and the axis of the channel at the edge was ~4°. The basic parameters were: d^* = 1.2 mm, d = 5.9 mm, corresponding to area ratios of 24.6 and 29.7. The length of the units in the direction of flow was 15 mm. The subsonic part was conical with a half-angle of 60°, and in the transonic region there was a cylindrical part 0.5 mm long.

2. Figure 1 shows dimensionless profiles for nozzle I of the total head $P_0' = p_0'/p_f$ (where p_0' is the total pressure downstream of the normal shock and upstream of the nozzle lip), the static pressure $P = p/p$, the values of the total pressure recovery coefficients $\sigma = p_0/p_f$ calculated with respect to p_0' and p in a cross section 20 mm high, and the Mach numbers M (points 1-4 correspond to $P_0' \times 10^2$, $P \times 10^3$, M, and $\sigma \times 10$ respectively). The measurements were performed along the axes of symmetry of the cross section. The coordinate y = 0 corresponds to the lower (wide) wall, and z = 0 to the side wall. The Pitot pressure in these experiments was p_Φ = 2.1 MPa.

The nonuniformity of the P(y) profile is related to the character of the streamlining of the corner point of the nozzle (see [8] for more details). The appreciable nonuniformity of the parameters along the z axis is a result of the side walls of the nozzle not being profiled. In this case a shock wave directed perpendicular to the wide wall is produced near the critical cross section [9].

For the experimental data the displacement thickness of the boundary layer on the wide wall of the channel was found to be δ^* = 0.87 mm. Calculation with the relation obtained from experiments with wind tunnel nozzles [10] gave δ^* = 0.64 mm. There is a considerable difference between the Mach numbers in the flow core and the values determined from the divergence ratio of the nozzle (M^0). Near the channel axis M ≈ 4.35, whereas M^0 = 5.13. The presence of shock waves in the flow leads to a large loss of total pressure (on the channel axis σ = 0.65).

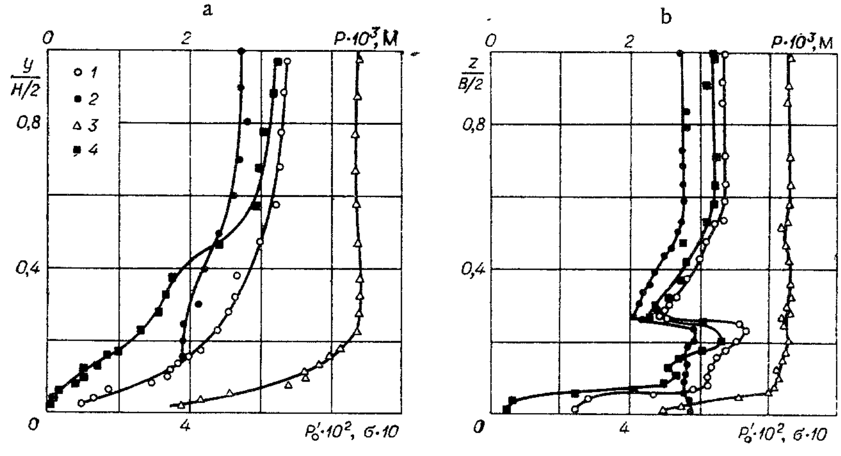

Fig. 1

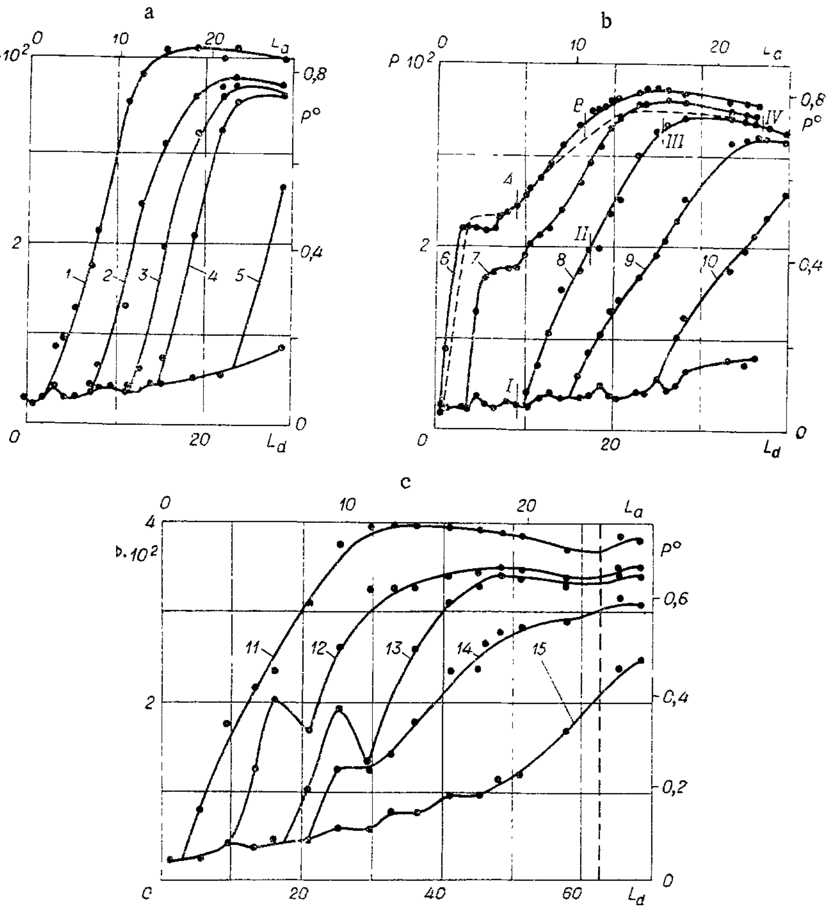

Fig. 2

A qualitatively similar pattern is observed also at the exit of nozzle II. The dimensionless pressure in the stream here is little more than half as large as in nozzle I, but the pressure loss is more substantial (at the center of the cross section $\sigma = 0.34$). Therefore, in spite of the considerable increase of the divergence ratio, the Mach number was little changed, and was 4.46 at the center of the cross section.

The distribution of the gasdynamic parameters in the channel downstream of a honeycomb nozzle is given in [11]. They are characterized by a substantial nonuniformity due to the necks between individual openings.

Using the experimental results shown in Fig. 1, we calculated the characteristics of the average flow with the same flow rate, momentum, and enthalpy as in nonuniform flow. A simplified method of averaging was used [12]. The calculation gave the following average flow parameters: $\langle P_0' \rangle_A = 5.00 \cdot 10^{-2}$ (for $M^0 = 5.13$, $P_0' = 5.42 \cdot 10^{-2}$), $\langle M \rangle_A = 3.90$, $\langle \sigma \rangle_A = 0.33$.

3. Using nozzle I, experiments were performed with channels having a ratio of sides B = 1.2, 3.8, and 8.9. The pressure distribution on the wall for these values of B is shown in Fig. 2a-c respectively (nozzles II and III were used only with the channel having B = 3.8). The ordinate on the left-hand side is the relative pressure on the wall, and the abscissa is the dimensionless length $L_d = l/d$, where l is the distance from the exit section of the nozzle, and d is the hydraulic diameter.

The experiments with the channels having B = 1.2 and 3.8 were performed at $p_f = 2.1$ MPa, and those for B = 8.9 at $p_f \approx 3$ MPa. Each curve in Fig. 2 corresponds to a definite output of the ejector.

Qualitatively the character of the $P(L_d)$ relation corresponds to the available results on the pressure variation in the pseudoshock obtained in cylindrical pipes. However, in channels with a large ratio of the sides, P does not always vary smoothly with L_d (curves 6, 7, 12-14). From all appearances the characteristic behavior of the pressure variation in these experiments for large B is related to flow separation. Earlier [6] flow separation was obtained in the entrance part of the pseudoshock in channels with B = 3-20, and asymmetry of the flow was noted. Three-dimensional flow continuing for a considerable distance was ob-

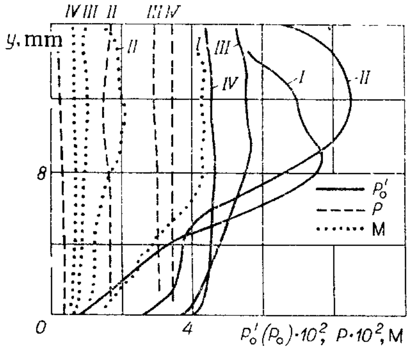

Fig. 3

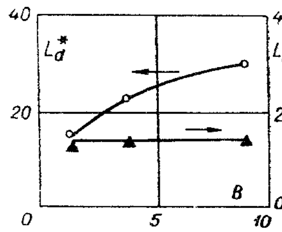

Fig. 4

served also in [5] for a channel with B = 8. Some of the experiments were performed in shortened channels with a length equal to the distance to sections A and B (Fig. 2b). The $P(L_d)$ relations obtained in these measurements agree completely with the pressure distribution along line 6 upstream of the corresponding cross sections (similar results were obtained in [3] under other conditions). Consequently, the relations obtained can be used also to determine the efficiency of diffusers shorter than those used in the present experiments.

The flow parameters in cross sections of the pseudoshock were measured with nozzle I (B = 3.8) at p_f = 2.1 MPa. The location of the cross sections in which the measurements were made is shown by curves I-IV of Fig. 2b. The results of the measurements and the processing of the experimental data are shown in Fig. 3.

4. Let us consider the question of the length l^* of the pseudoshock, equal to the distance from the beginning of the sharp rise to the cross section at which the pressure is maximum. Because the peak is rather flat, it is difficult to determine l^* accurately. On the basis of the experimental data shown in Fig. 2 (curves 1-4, 6-9, 11-14) the relative length of the pseudoshock is $L_d^* = l^*/d = 15.5 \pm 1.5$, 23 ± 2, and 35 ± 5 for B = 1.2, 3.8, and 8.9, respectively (curves 5 and 10 do not reach a maximum within the limits of the experimental part). Under similar conditions $L_d^* = 13$ in a cylindrical pipe [1]. The dependence of L_d^* on B is shown in Fig. 4. A diffuser of constant cross section can be shortened without significantly decreasing its efficiency. For example, a pressure equal to 0.95 of maximum is realized at a distance $L_d^*(0.95) = 10.5 \pm 1$, 17 ± 1, and 24 ± 3 for B = 1.2, 3.8, and 8.9, respectively.

It is known [1] that L_d^* depends on the average Mach number M_1 in the entrance section of the pseudoshock. In the present experiments M_1 was varied over the limits ~4-3. A definite dependence of L_d^* on M_1 could not be determined because of the flat peak. It can only be noted that the upper limit of L_d^* generally corresponds to a larger M_1. The values of L_d^* presented above include the results of experiments with nozzles II and III also.

The height h and the width b of a rectangular channel (B = b/h > 1) do not have the same effect on the length of the pseudoshock. On the basis of the experiments performed, the following empirical relation was obtained:

$$L_a^* = l^*/a = 14, \qquad (4.1)$$

where the effective dimension $a = h^{0.3}b^{0.7}$. The functions L_a^* (B) and L_d^*(B) are shown in Fig. 4, and in Fig. 2 the upper scale of abscissas is the dimensionless distance $L_a = l/a$.

For h = b the formula gives a value close to the length of the pseudoshock in cylindrical pipes [1]. For the other limiting case ($b/h \to \infty$) Eq. (4.1) gives a physically wrong value of L_a^*. However, the range B = 1.2-8.9 studied is quite sufficient for many practical applications.

Experiments were performed to shorten the slowing-down region. In the experiments with nozzle I (B = 3.8) this was done by placing grid I of 14 cones at a distance L_d = 1.6. In another experiment grid II with eight cones was placed in the section L_d = 13.6. Details of the construction of the grids and the experimental results are shown in Fig. 5.

It is clear that the pseudoshock was shortened by a factor of 2-4 when grid I was located in the section L_d = 1.6 (Fig. 5, curves 1 and 2). Simultaneously the maximum efficiency of the pressure recovery was decreased by approximately 10-20% (the open curves are plotted from data of Fig. 2b for unperturbed flow).

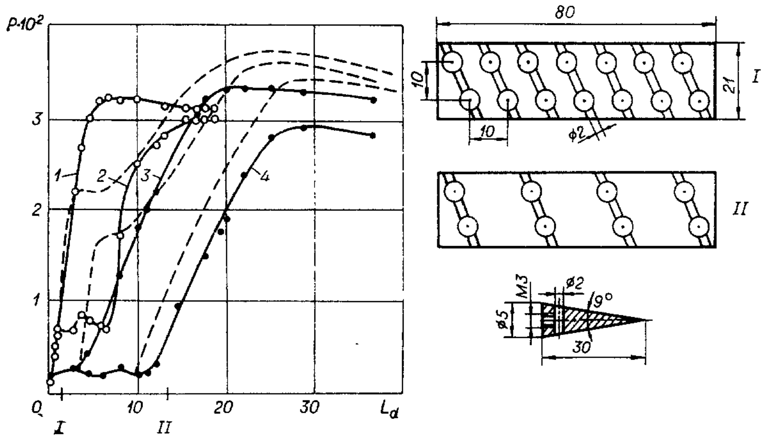

Fig. 5

Placing the grid of eight cones in the section L_d = 13.6 has an appreciably smaller effect on the length of the pseudoshock (Fig. 5, curves 3 and 4); L_d^* was decreased by ~30%. This result can be accounted for by the fact that the weakly disturbed core flow is small upstream of the grid. Therefore, the additional perturbations from the cones are not very effective.

5. Before turning to an analysis of the efficiency of a diffuser of constant cross section, we determine the range of its variation. As in [1], we replace the pseudoshock by a normal shock with parameters upstream of it calculated from the equations of conservation of mass and energy of adiabatic flow. As a result we obtain an equation for the average parameters downstream of the pseudoshock (the flow in the critical section is assumed one-dimensional):

$$p_{00}A^* = p_{01}Aq(\lambda_1) = p_{02}Aq(\lambda_2). \tag{5.1}$$

Here p_{00} is the initial total pressure, p_{01} and p_{02} are the average total pressures upstream and downstream of the pseudoshock (henceforth the subscripts 1 and 2 will correspond to the average parameters in these sections), λ is the velocity coefficient, $q(\lambda) = ((k+1)/2)^{1/(k-1)}\lambda(1-\lambda^2(k-1)/(k+1))^{1/(k-1)}$; k is the adiabatic exponent, and A* and A are the areas of the critical cross section and the channel respectively. It follows from (5.1) that

$$\sigma = \frac{p_{02}}{p_{00}} = \frac{A^*}{A}\frac{1}{q(\lambda_2)} = \frac{q(\lambda^0)}{q(\lambda_2)}, \tag{5.2}$$

where λ^0 is the geometric value of the velocity coefficient at the nozzle exit, defined as A/A* and corresponding to an ideal expansion of the gas (henceforth a superscript 0 corresponds to the parameters of such a flow). In the absence of friction and heat transfer, the value of $\sigma^0 = p^0_{02}/p_{00}$ is found from the relation for the normal shock

$$\sigma_0 = q(\lambda^0)/q(1/\lambda^0). \tag{5.3}$$

Dividing (5.2) by (5.3) and using the relation $\lambda_2 = 1/\lambda_2$ for the normal shock as applied to the average parameters, we find the ratio of the total pressure p_{02} downstream of the shock to the maximum possible value p^0_{02}:

$$\eta^0_0 = \frac{\sigma}{\sigma^0} = \frac{p_{02}}{p^0_{02}} = \frac{q(1/\lambda^0)}{q(1/\lambda_1)}. \tag{5.4}$$

The analogous ratio of the static pressures has the form

$$\eta^0 = \frac{p_2}{p^0_2} = \frac{y(1/\lambda^0)}{y(1/\lambda_1)}, \tag{5.5}$$

where

$$y(\lambda) = ((k+1)/2)^{1/(k-1)}\lambda/(1-\lambda^2(k-1)/(k+1)).$$

An approach close to but not identical with that described was used in [10, 13] to analyze the loss in a channel of supersonic wind tunnels. The total pressure recovery coefficient under conditions typical for such equipment turned out to be close to σ^0 (cf. (5.3)). In those same papers a parametric analysis of the effect of the boundary layer thickness at the exit cross section of a supersonic nozzle showed that even for a large change of the Mach

TABLE 2

M°	$\eta^0_{0\,\min}$	$\eta^0_{\min}$	M°	$\eta^0_{0\,\min}$	$\eta^0_{\min}$
3	0,72	0,44	10	0,61	0,36
5	0,65	0,38	∞	0,60	0,32

number in the flow core the increase in the loss of total pressure in comparison with (5.3) is several percent for M° = 10-18. The main result derived in [10, 13] follows from (5.4) as $\lambda_1 \to \lambda^\circ$.

In a latter experiment in a wind tunnel the deviation from this rule turned out to be small even for $M^\circ = 3.7$ [14]. However, the possibility of the extension of the results in [10, 13] to the supersonic range of velocities and an arbitrarily long channel was not investigated. Therefore, let us analyze more rigorously the limits of variation of the losses during slowing down of a supersonic flow in a channel of constant cross section.

The average value of the velocity coefficient in the entrance section of the pseudoshock can vary over the range $\lambda^\circ \leqslant \lambda_1 \leqslant 1$. For $\lambda_1 = \lambda^\circ$ the efficiency coefficients of the pseudoshock reach the largest value $\eta^\circ_\circ = \eta^\circ = 1$. In the other limiting case ($\lambda_1 = 1$) the slowing down efficiency in the pseudoshock is minimum. The values of $\eta^\circ_{\circ\min}$ and $\eta_{\min}$ are listed in Table 2 as functions of the geometric Mach number M°.

The efficiency of the pressure recovery $\eta^\circ_\circ = \eta^\circ \simeq 1$ is realized physically in a channel of equipment with a well-profiled nozzle at large Re, and the beginning of the pseudoshock near the exit section of the nozzle when the relative displacement thickness of the boundary layer δ^*_1/d is small. On the other hand, the minimum values $\eta^\circ_{\circ\min}$ and $\eta^\circ_{\min}$ correspond to a small Re and/or a displacement of the pseudoshock far downstream. Table 2 shows that, within the framework of the model considered, losses related to viscous effects do not change the total pressure in the pseudoshock very much even in the limiting case $M^\circ \to \infty$, $M_1 = 1$ ($\eta^\circ_{\circ\min}$ = 0.60).

Extremely small values of $\eta^\circ_{\min}$ were not reported in the papers with which we are familiar because of the small values of the relative length of the channel (for corresponding Re). This case was realized in the present experiments in the channel with B = 8.9 (curves 15 of Fig. 2c). The average Mach number was determined at the end of this channel (denoted by a vertical dashed line). This was done as in [1] by using the relation

$$y(M_1) = \frac{1}{P_1}\frac{1}{A/A^*}, \tag{5.6}$$

which follows from the equations for the conservation of mass and energy under the assumption that P_1 is constant over the cross section of the channel. Calculation gives the value $M_1 \approx 1$. The increase in pressure after a small expansion of the flow (to the right of the dashed line) shows that in the cross section under consideration the flow on the average is most likely subsonic. In addition, by using (5.5) the value of the minimum relative pressure recovery in the pseudoshock was found to be $\eta^\circ_{\min} = 0.385$, which is in good agreement with the experimental value $\eta^\circ_{\min} = 0.39$. This agreement shows that the assumptions used in deriving (5.5) and (5.6) are realistic.

Now let us consider how greatly the pressures reached in the experiments differ from the maximum possible. To do this we again return to Fig. 2 where the values of the ordinate $P^\circ = p/p^\circ_2$ are read on the right-hand side (in the calculation of p°_2 for the channels with B = 1.2 and 8.9 the reduction of the cross section by the diaphragms was taken into account). The largest values of η° (P° at the point of the maximum) in our experiments were 0.86, 0.82, and 0.76 for B = 1.2, 3.8, and 8.9 respectively. The maximum efficiency of the total pressure recovery was determined in experiments with nozzle I (B = 3.8), when the beginning of the pseudoshock was found immediately downstream of the exit cross section of the nozzle, and the measurements were performed at a distance $L_d = 36$. The average over the area $\langle p_{o2\max}\rangle = 4.65\cdot 10^{-2}$ corresponds to $\eta^\circ_\circ = 0.9$.

Thus, in spite of the large losses in the nozzle, and the appreciable difference between the average Mach number and the calculated value, the maximum efficiency of the pressure recovery in the pseudoshock is close to the limit for the given expansion ratio of the nozzle. This is in accord with (5.4) and (5.5), since in this situation λ_1 is little different from λ°. As noted earlier, a similar conclusion was drawn in [10, 13] on the basis of a computational analysis of the effect of the boundary layer in a supersonic nozzle on the value of σ.

TABLE 3

No. of curves in Fig. 2	η	η'	No. of curves in Fig. 2	η	η'
1	0,91	0,96	7	0,82	0,88
3	0,92	0,99	8	0,80	0,88
6	0,86	0,92	11	0,81	0,88

TABLE 4

Reference	A/A^*, (M^0)	B	L'_d	η'
[6]	16,56	9,08	9	0,47
		3	12	0,67
[4], Fig. 5, 6	19,3	7,65	6,4	0,7
[5]	(3)	8	12	0,65 †
[7]	(5)	4,5	15	0,52 †
[3], Fig. 8	(3)	2	15	0,58—0,77 ‡

† $p_{max}/p_2(M^0)$, ‡ η.

In experiments with nozzles II and III, which differ from nozzle I in the profile of the subsonic and supersonic parts and the expansion ratio, the values obtained for the efficiency coefficients were close to the maximum values indicated above. Thus, for nozzle II $\eta^0 = 0.90$ and $\eta_0^0 = 0.95$. The open curve in Fig. 2b shows $P^0(L_d)$ for a set of conical nozzles. In this case $\eta^0 = 0.78$ and $\eta_0^0 = 0.80$. For comparison we present also the value of η_{max}^0 for a cylindrical pipe. Under the conditions of the experiments in [1] $\eta_{max}^0 = 0.90$. On the basis of the results obtained it can be stated that the method of profiling and even the method of obtaining supersonic flow (nozzle or nozzle sets) in the range of A/A^* values studied affect the losses in the nozzle structures themselves and determine the distribution of parameters in the exit cross sections, but only slightly change the maximum efficiency of the pressure recovery. The shape of the channel cross section also hardly changes the values of η_{max}^0.

Nevertheless, the coefficients η_0^0 and η^0 are not a sufficiently general characteristic of the pseudoshock, since they determine the pressure recovery efficiency of the nozzle-diffuser system. To compare the efficiencies of the pseudoshock itself obtained under various experimental conditions it is more logical to use the parameter

$$\eta = p_2/p_{21}(M_1), \tag{5.7}$$

where p_2 is the maximum pressure in the channel for an arbitrary position of the beginning of the pseudoshock relative to the nozzle edge (cross section L_1); p_{21} is the calculated pressure downstream of the normal shock for $M = M_1$ and $p = p_1$. The Mach number in the cross section L_1 is found from Eq. (5.6) for known P_1 and A/A^*.

In addition, in (5.6) we take account of the coefficient of discharge μ of the nozzle:

$$y(M_1) = \mu(1/P_1)(A/A^*)^{-1} \tag{5.8}$$

and introduce the parameter

$$\eta' = p_2/p'_{21}(M_1), \tag{5.9}$$

where $p'_{21} = p_{21} - \Delta p$. Calculations showed that the pressure decrease Δp in the pseudoshock as a result of friction is relatively small, at least for $Re_d \geqslant 10^4$. This is confirmed experimentally [16]. Therefore, we estimated Δp from data for stabilized flow in pipes, and calculated Re_d from flow parameters downstream of the normal shock in the entrance section of the pseudoshock. Such a result is given in [2].

The values of η and η' calculated from (5.6), (5.7) and (5.8), (5.9) respectively are listed in Table 3 for experiments with nozzle I ($Re_d = (2\text{-}3)\cdot 10^5$). It was assumed that $\mu = 1$. Nearly the same results were obtained also for a larger expansion ratio (nozzle II): $\eta = 0.89\text{-}0.94$, $\eta' = 0.94\text{-}1$. We note here that $\eta = 0.95\text{-}0.99$ for cylindrical pipes also. The deviation of η' from unity is related to the assumption $\mu = 1$, to the approximate method of taking account of friction, to other assumptions of the analysis, and to the accuracy of the experiment. However, on the whole it can be stated that this characteristic of the pseudoshock is unaffected by a change of shape, the expansion ratio of the nozzle, the configuration of the channel, and the prehistory of the flow.

On the other hand, the length of the pseudoshock is to a greater extent affected by different factors. This, in particular, is shown by the results of the present experiments and those in [3, 16].

Taking account of what has been said about the parameter η', we analyzed the data in the literature on the efficiency of the pseudoshock in rectangular channels. The conditions of the experiments are given in Table 4.

In all the papers examined η' was smaller than in a cylindrical pipe and in a rectangular channel under conditions of the present experiments. In our opinion the main reason for the low efficiency of the slowing down of the flow in the investigations analyzed is the inadequate length of the channel. With the exception of [3], it turned out to be shorter than the length of the pseudoshock established in the present experiments (Fig. 4). In this case the mechanism of the pseudoshock is only partially realized (cf. points A and B in Fig. 2b). The following can be said relative to [3] where, in spite of $L > L^*$, $\eta \ll 1$. To all appearances the author used a different method of determining η, or simply calculated the pressure ratio incorrectly. Using the input data of Fig. 8 in [3], we obtained $\eta' = 0.98$-0.99, which is in complete agreement with our experiments.

Thus, as a result of our research we found the dependence of the length of the pseudoshock in a rectangular channel on the ratio of the sides for $M_1 = 3$-4. When the length of the channel is greater than the values given by Eq. (4.1), the maximum pressure recovery is close to its value in a circular pipe. The low efficiency of rectangular diffusers in the previous research is related mainly to the inadequate length of the experimental channel.

The magnitude of the efficiency coefficient of the pseudoshock ($\eta' = 0.9$-1) does not depend on the expansion ratio, the shape of the nozzle, the configuration of the channel, or the prehistory of the flow. Thus, our results enable us to choose the optimum length and to determine the pressure at the exit from a rectangular diffuser of constant cross section for $B \leqslant 9$ and $M_1 = 3$-4.

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DEVELOPMENT OF INTERNAL WAVES GENERATED BY A CONCENTRATED PULSE SOURCE IN AN INFINITE UNIFORMLY STRATIFIED FLUID

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1. The question of the fundamental function of the internal wave operator (FFIWO) in an infinite incompressible uniformly stratified fluid has been raised several times in recent years (see [1-4], say). This is natural: Its solution is of top priority in construction of a theory of linear forced internal waves (FIW). Almost everything is now known about FFIWO; in particular, a representation has been found in the form of a single integral in the frequencies, and its asymptotic has been obtained for large values of the time. Available information about the field of unsteady FIW is not detailed enough, however; there is no information about the limits of applicability of the asymptotic formulas, and no complete clarity relative to the initial stage of wave field evolution. The gap can be eliminated by using a numerical computation, but this has not yet been done, and this paper intends to fill this gap. Its main content is the elucidation of results of a computer computation of the integral representation for an unsteady FIW field, and their comparison with a computation of the asymptotic for this field in order to determine its range of action.

The FFIWO has no direct physical meaning; hence, the main attention will not be turned to this function. From the physical viewpoint, the consideration of the wave function of a concentrated pulse source (CPSF) is expedient (especially if we have in mind the application to problems of stratified fluid flow around bulk bodies). It was not apparently studied in detail earlier. Our computations refer precisely to the case of the action of a concentrated pulse source in an infinite incompressible uniformly stratified fluid. Since both functions, the FFIWO and CPSF, are closely interrelated, appropriate deductions relative to the properties and behavior of the fundamental function can be made from these computations. There are certainly distinctions. They will be mentioned below, together with the similarity in the behavior of these functions.

2. As the CPSF we take the vertical displacement of a fluid particle due to the FIW. We denote it by $\zeta = \zeta(x, y, z, t)$. Here x, y, z are the two horizontal and one vertical coordinate of the observation point (the z axis is directed upward), and t is the time. The origin of the right-handed rectangular coordinate system is set at the location of the pulse source whose action occurs instantaneously at the time t = 0 and causes the appearance of the FIW. We consider that an incompressible fluid fills the whole space and is stratified uniformly along the vertical. The Weisailla frequency therein is constant and equal to N. The examination is performed within the framework of linear theory and the Boussinesq approximation.

The equation of FIW dynamics has the following form in this case

$$\left(\partial_t^2\left(\partial_x^2+\partial_y^2+\partial_z^2\right)+N^2\left(\partial_x^2+\partial_y^2\right)\right)\zeta(x, y, z, t)=Q\partial_z\partial_t\delta(x, y, z, t), \tag{2.1}$$

where Q is the total debit of the source of the mass, ∂_x, ∂_y, ∂_z, ∂_t are differentiation operators with respect to x, y, z, t, respectively, and $\delta(x, y, z, t)$ is the Dirac function. The quantity ζ satisfies the causality condition

$$\zeta=0 \quad \text{for} \quad t<0 \tag{2.2}$$

(in the sense of the theory of generalized functions [5]).

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